Policy Based Inference in Trick-Taking Card Games

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***Abstract*—Trick-taking card games feature a large amount of private information that slowly gets revealed through a long sequence of actions. This makes the number of histories exponentially large in the action sequence length, as well as creating extremely large information sets. As a result, these games become too large to solve. To deal with these issues many algorithms employ inference, the estimation of the probability of states within an information set. In this paper, we demonstrate a Policy Based Inference (PI) algorithm that uses player modelling to infer the probability we are in a given state. We perform experiments in the German trick-taking card game Skat, in which we show that this method vastly improves the inference as compared to previous work, and increases the performance of the state-of-the-art Skat AI system Kermit when it is employed into its determinized search algorithm.**

***Index Terms*—Game AI, Inference, Card Game, Neural Networks, Policy Learning, Skat**

I. Introduction

Determinized search algorithms allow for the application of perfect information algorithms to imperfect information games. While this may not always be a good idea, in some cases it represents the current state-of-the-art. These algorithms are composed of two steps: sampling and evaluation. First, a state is sampled from the player's current information set; informally, an information set is a set of states that a player cannot tell apart given their observations. After a state is sampled, it is evaluated using a perfect information algorithm such as minimax.

Inference is a central concept in imperfect information games. It involves using a model of the opponent's play to determine their hidden information based on the actions taken in the game so far. Because the states that constitute the player's information set are not always equally likely, inference plays a key role in the performance of determinized search algorithms.

While counter-factual regret (CFR) techniques have produced super-human AI in Poker [1], [2], they have not proven useful for trick based games. This is due to the extremely large size of the information sets, the long length of bidding and cardplay sequences, and the difficulty in creating expressive abstractions. While these long sequences make the game too large to solve, they also slowly reveal the private information of the other players, thus making inference a desirable approach.

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In this paper, we show how an opponent model can be used for inference in trick-taking card games. In particular, we train policies on supervised human data and use them to infer the private information of opponents and partners based on each of their previous actions. This leads to improvements over the previous state-of-the-art techniques for inference in the domain of Skat.

The rest of this paper is organized as follows. First, we explain the basic rules of Skat and then work related to inference in trick-taking card games. Next, we outline an algorithm for performing inference in trick-taking card games using an opponent model trained on data from a diverse set of human players which we term Policy Inference (PI). This algorithm assumes a policy of the opponents, and directly estimates the reach probability of a sampled state by computing the product of all probabilities of the actions in the history conditioned on that sampled state. We evaluate this algorithm empirically in Skat and show that it significantly outperforms previous work both in tournament settings and at selecting the true underlying state. Finally, we conclude the paper and provide ideas for future research.

II. Background

Trick-taking card games, like Contract Bridge, Skat, and Hearts, are imperfect information games in which information set sizes shrink rapidly due to hidden information being revealed by player actions. Long et al. [3] explain why this is an appropriate setting for determinized search algorithms such as Perfect Information Monte Carlo [4] and Information Set Monte Carlo Tree Search [5]. These algorithms are considered state-of-the-art in several trick-taking card games, including Bridge [6] and Skat [7].

After sampling, states are evaluated using perfect information evaluation techniques, but this can be problematic. In perfect information game trees, the values of nodes depend only on the values of their children, but in imperfect information games, a node's value can depend on other parts of the tree. This issue, called non-locality, is one of the main reasons why determinized search has been heavily criticized in prior work [8], [9]. Inference helps with non-locality by biasing state samples so that they are more realistic with respect to the actions that the opponent has made. This seems to improve the overall performance of determinized algorithms. However, the gains provided by inference come at the cost of increasing the player's exploitability. If the inference model is incorrect or has been deceived by a clever opponent, using it can result in low-quality play against specific opponents.

A. Related Work

Previous applications of determinized search in trick-taking card games acknowledge the relationship between inference and playing performance. The first successful application of determinized search in a trick-taking card game was GIB [6] in Contract Bridge. The author suggests that only deals consistent with the actions taken so far are sampled for evaluation. More specific details are not provided. WBridge5 [10] and Jack [11] have had recent success in the World Computer Bridge Championship [12], but their implementation details are not readily available.

In Skat, Kermit [7], [13] used a table-based technique to bias state sampling based on opponent bids and declarations. This approach only accounts for a limited amount of the available state information and neglects important inference opportunities that occur when opponents play specific cards. This inference will be referred to as Kermit Inference (KI) for the rest of the paper. Solinas et al. [14] extend this process by using a neural network to make predictions about individual card locations. By assuming independence between these predictions, the probability of a given configuration was calculated by multiplying the probabilities corresponding to card locations in the configuration. This enables information from the card-play phase to bias state sampling. While this method is shown to be effective, the independence assumption does not align with the fact that for a given configuration, the probability that a given card is present is highly dependent on the presence of other cards. For instance, their approach cannot capture situations in which a player's actions indicate that their hand likely contains either the clubs jack or the spades jack, but not both. The Policy Inference approach presented in this paper captures this context by estimating a state's probability based on the exact card configuration of that state. In this way, more precise inference is possible even though both techniques use the same body of data. The major upside of the Card Location Inference (CLI) is that it runs much faster than the Policy Inference.

In other domains, Richards and Amir [15] model the opponent's policy using a static evaluation technique and then perform inference on the opponent's remaining tiles given their most recent move in Scrabble. Baier et al. [16] leverage policies trained from supervised human data to bias MCTS results; this is similar to our approach in that it uses human data to train an opponent model, but this model is not used to infer opponent hidden information. Sturtevant and Bowling [17] build a generalized model of the opponent from a set of candidate player strategies. Our use of aggregated human data could be viewed as a general model that captures common action preferences from a large, diverse player base.

B. Skat

Our application domain for this paper is the game of Skat. It is a 3-player trick-taking card game that originates in Germany in the 1800s and is played competitively around the world. Skat is played using a 32 card deck where cards 2 through 6 from each suit are removed from the standard 52 card deck.

TABLE I: Game Type Description

Type	Base Value	Trumps	Soloist Win Condition
Diamonds	9	Jacks and Diamonds	$\geq$ 61 card points
Hearts	10	Jacks and Hearts	$\geq$ 61 card points
Spades	11	Jacks and Spades	$\geq$ 61 card points
Clubs	12	Jacks and Clubs	$\geq$ 61 card points
Grand	24	Jacks	$\geq$ 61 card points
Null	23	No trump	losing all tricks

TABLE II: Game Type Modifiers

Modifier	Description
Schneider	$\geq$90 card points for soloist
Schwarz	soloist wins all tricks
Schneider Announced	soloist loses if card points $<$ 90
Schwarz Announced	soloist loses if opponents win a trick
Hand	soloist does not pick up the skat
Ouvert	soloist plays with hand exposed

Play starts after each player is dealt 10 cards; the two that remain are called the "skat", and are placed face down in the middle. After observing their cards, players engage in the bidding phase to see which of them play against the other two in the subsequent cardplay phase. In the bidding phase, players alternate making successively higher bids based on their hand and the highest-valued game they believe they could win. This value is dependent on the game type (see Table I) and a multiplier that is based on the cards in the player's own hand and the outcome of the game (see Table II).

Once the highest bidder is determined, that player has the option of picking up the skat and discarding any two cards from their hand. The same player declares a game type, which determines the specific rules used in the upcoming cardplay phase — including the win condition and which suit will be trump.

As in other trick-taking card games, the cardplay phase revolves around winning tricks. Tricks start with the trick leader playing a card and proceed in clockwise order. Players must play a card from the same suit as the card that was initially played by the leader if they have one. Otherwise, any card can be played. After every player has played a card, the highest ranked card of either the led suit or the trump suit (if a trump was played) wins.

All game types involve the "soloist" (the player who won the bidding) playing against a team of "defenders" (the other two players). In suit and grand games, both parties receive points for winning tricks containing certain cards. The soloist is required to amass at least 61 out of the possible 120 points to win the game. In null games, the soloist must lose every trick to win. The soloist's score is either increased by the game value if the game was won, or decreased by double the game value if it was lost. Players play a sequence of 36 hands and keep a tally of the score over all hands to determine the overall winner in the competitive setting.

III. INFERENCE

To determine the probability of a given state s in an information set I, we need to calculate its reach probability η. If we can perfectly determine the probability of each action that leads to this state, we can simply multiply all the probabilities together and get η. Each s in I has a unique history h, the sequence of all previous s and a that lead to it. $h \cdot a$ represents the history appended with the action action taken at that state. Thus, there is a subset of h, containing all the $h \cdot a$ for a given s. Formally:

$$\eta(s|I) = \prod_{h \cdot a \sqsubseteq s} \pi(h, a) \quad (1)$$

For trick-taking card games, the actions are either taken by the world (chance nodes in dealing), other players' actions, and our actions. Transition probabilities of chance nodes can be directly computed since these are only related to dealing, and the probability of our actions can be taken as 1 since we chose actions that lead to the given state with full knowledge of our own policy. This leaves us with determining the move probability of the other players. If we have access to the other players' policies, we can use Equation (1) to perfectly determine the probability we are in a given state within the information set. If we repeat this process for all states within the information set, we can calculate the probability distribution across states. If we can perfectly evaluate the value of all the state-action pairs, we can select the action that maximizes this expected value which provides an optimal solution.

There are two main issues with this approach. The first is that we either do not have access to the other players' policies, or they are expensive to compute. This makes opponent/partner modelling necessary, in which we assume a computationally inexpensive model of the other players, and use them to estimate the reach probability of the state. The second problem is that the number of states in the information set can be quite large. To get around this, we can sample the worlds and normalize the distribution over the subset of states. Because in Skat the information set size for a player prior to card-play can consist of up to 2.8 billion states we employed sampling. We use Algorithm 1 to estimate a state's relative reach probability. When we do not sample, this becomes an estimate of the states true reach probability.

While we have access to the policy of the current strongest Skat bot, using its policy directly would be computationally intractable because it uses an expensive search based method that also performs inference. Also, it is the goal of this research to develop robust inference that is not based upon the play of a single player. Thus, we decided to use policies learned directly from a large pool of human players. These policies are parameterized by deep neural networks trained on human games [18]. The features for the pre-cardplay networks are a lossless one-hot encoding of the game state while considerable feature engineering was necessitated for the cardplay networks. Separate networks were trained for each distinct decision point in the pre-cardplay section, and for each game type for the cardplay networks. More details on the training and the dataset can be found in [18].

```
EstimateDist(InfoSet I, int k, OppModel π)
    S ← SampleSubset(I, k)
    for s ∈ S do
        η(s) ← 1
        for h, a ∈ StateActionHistory(I) do
            η(s) ← η(s) * π(h, a)
        end
    end
    return Normalize(η)
```

Algorithm 1: Estimate the state distribution of an information set given an opponent model and the actions taken so far.

The decision points in pre-cardplay are bidding, picking up or declaring a hand game, choosing the discard, and declaring the game. The decision points in the cardplay section are every time a player chooses what card to play. While inference would be useful for decision-making in the pre-cardplay section, we are only applying it to cardplay in this paper. As such, we can abstract the bidding decisions into the maximum bids of the bidder and answerer in the bid/answer phase, and the maximum bids of bidder and answerer in the continue/answer phase. For these maximum bid decision points, we only observe the maximum bid if the player passes. For the cases in which the intent of maximum bid is hidden, the probability attached to that decision point is the sum of all actions that would have resulted in the same observation, namely the probability of all maximum bids greater than the pass bid. The remaining player decision points are pickup or declare a hand game, discard and declare, and which card to play. As these are not abstracted actions, the probability of the move given the state can be determined directly from the appropriate trained network.

The current state-of-the-art Skat bot, Kermit, uses search-based evaluation that samples card configurations. A card configuration is the exact location of all cards, and thus doesn't take into account which cards where originally present in the soloist's hand prior to picking up the skat. Depending on the game context, there are either 1 or 66 (12 choose 2) states that correspond to a card configuration during the cardplay phase. Two variants of inference were explored. The first variant samples card configurations. Decision points are ignored for inference if there are multiple states with the same card configuration but different features (input to the network). The second variant samples states directly, thus avoiding this issue. For our implementation, the need to distinguish states that share a configuration only occurs when a player does inference on the soloists actions prior to picking up the two hidden cards in the skat. Sampling card configurations will be treated as the default approach for PI. When states are sampled instead, the inference will be labelled PIF, for Policy Inference

Full.

IV. Experiments

In this section, we test the quality of the inference directly and indirectly through the players' performance in a tournament setup. The baseline players are all versions of Kermit, with the only difference being the inference module used. These inference modules are the original Kermit Inference (KI) [13], card-location inference (CLI) [14], and no inference (NI).

A. Direct Inference Evaluation

To measure the inference quality directly, we measured the True State Sampling Ratio (TSSR) [14] for each main game type, separately for defender and soloist. TSSR measures how many times more likely the true state will be selected than uniform random.

$$TSSR = \eta(s^*|I) \ / \ (1/|I|) = \eta(s^*|I) \cdot |I| \qquad (2)$$

$\eta(s^*|I)$ is the probability that the true state is selected given the information set I, and $|I|$ is the number of possible states. Since the state evaluator of Kermit does not distinguish between states within the same card configuration, we will slightly change the definition to measure how many more times likely the card configuration (world) will be sampled than uniform random.

Since the players tested use a sampling procedure when the number of worlds is too large, the $TSSR$ value cannot be easily computed directly as this would require all the world probabilities to be determined. We therefore estimate it empirically. Since sampling was performed without replacement, we use the given inference method to evaluate η given that the true world was sampled k times. We can combine these values to get the combined probability that the true world is selected:

$$TSSR = |I| \cdot \Sigma_k BinDist(k, p) \cdot k \cdot \eta(s^*|I, k) \qquad (3)$$

where $BinDist$ is the probability mass of the binomial distribution with k successes and probability of sampling the true world p which is $1/|I|$. Terms of the summation were only evaluated if the $BinDist(k, p)$ value were significant, which we cautiously thresholded at 10^{-7}.

When the number of worlds is less than the set threshold parameter specific to the player, we sample all worlds and can directly compute the value. For the sake of the TSSR experiments, null games were further subdivided into the two main variants, null and null ouvert. Null ouvert is played with an open hand for the soloist, thus making the inference quite different from that of regular null games from the perspective of the defenders. For each game in the respective test set, the TSSR value was calculated for each move for the soloist, and one of the defenders. The test set was taken from the human data, and was not used in training. The number of games in the test sets were 4,000 for grand and suit, 3,800 for null, and 13,000 for nul ouvert.

Figure 1 shows the average TSSR metric after varying number of cards have been revealed. The inference variants tested are PI20, PIF20, PI100, CLI, and KI. NI was not tested because it will always have a value of 1. PI20 and PI100 sample 20,000 and 100,000 card configurations respectively, while PIF20 samples 20,000 states. CLI samples 500,000 card configurations, and KI samples 3200 card configurations in soloist, and a varying number in defense. CLI inference was not implemented for null ouvert.

TSSR is higher on defense, with the exception of null ouvert. This is likely due to there being many more possible worlds in the defenders information set because of the hidden cards in the skat. Also, the defender can use the declaration of the soloist for inference, which is a powerful indicator of the soloist's hidden cards. Null ouvert does not follow this trend because there are only 66 possible worlds at most in defense while there are 184,756 for the soloist. This allows for higher TSSR values for the soloist.

PI20, PIF20, and PI100 all achieve significantly higher TSSR values than the other methods, across all game-types and roles. KI performs better than CLI at the beginning of games, but surpasses KI once more cards are played.

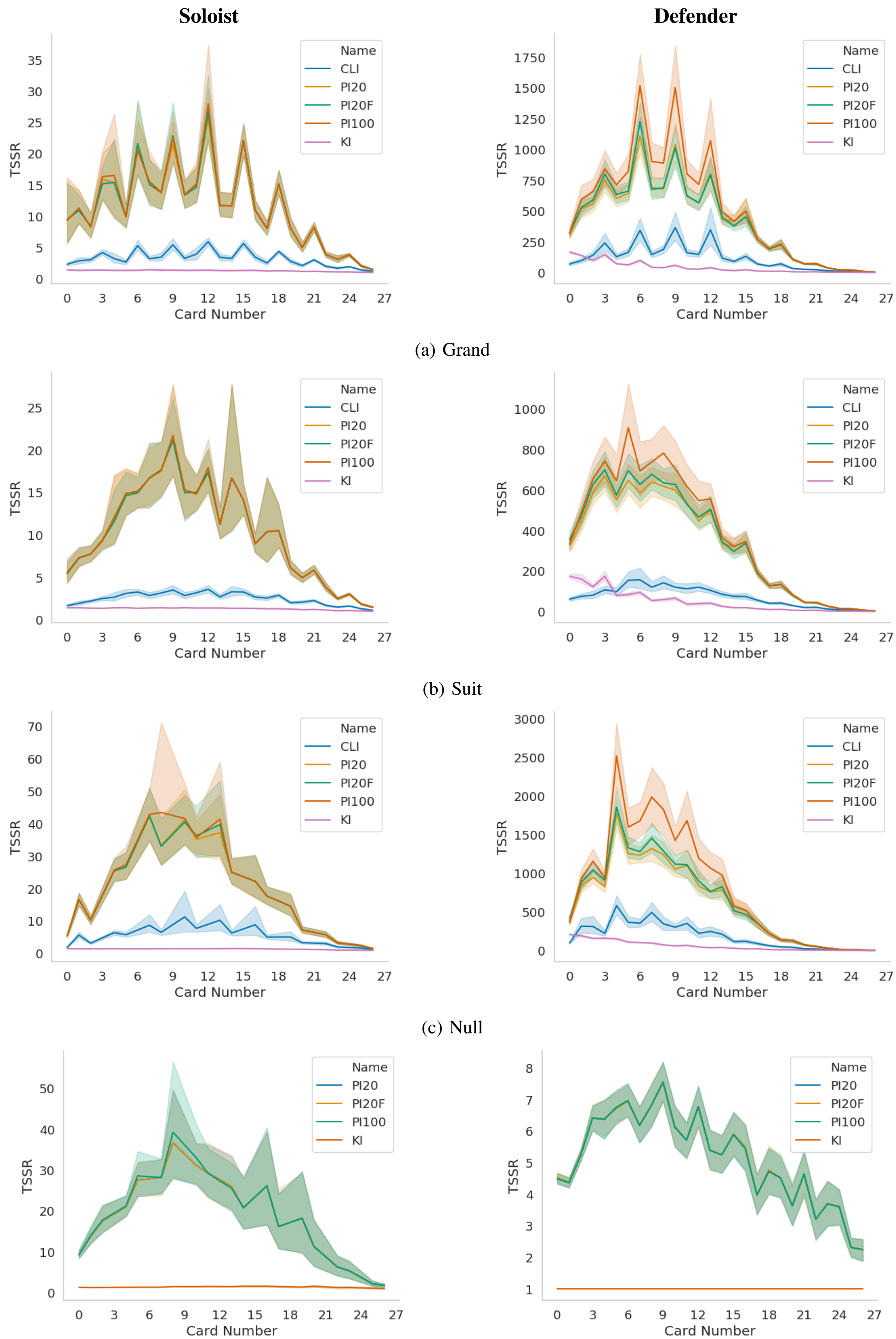
PI100 appears to consistently perform better in defense than the other Policy Inference variants, while PIF20 appears to perform slightly better than PI20 in the first half of defender games, but not significantly so. All TSSR values trend down to 1 at the endgame, as the number of possible worlds approaches 1.

One common feature across all games is the spiking of TSSR values, which is best exemplified in suit games. The spiking is consistent between players within the same game type and role graph. However, between graphs it is not consistently occurring at the same number of cards played. We do not see an obvious reason for this. However, these tests were done on human games and thus we are not controlling for inherent biases in the distribution. Further investigation is needed to determine why these spikes occur.

It is clear from these results that the Policy Inference approach provides larger TSSR values, since the error envelopes are completely separated in the figure. It also should be noted that perfect inference would not result in the upper bound TSSR value which is equal to the number of worlds. Even with perfect knowledge of the opponents' policies, uncertainty is inherent and thus a player with perfect TSSR value is not possible.

B. Cardplay Tournament

To test the performance of PI in cardplay, we played 5,000 matches for each of suit, grand, and null games against baseline players in a pairwise setup. Only the cardplay phase of the game is played, while the bidding and declaration is taken directly from the human data-set. These games were held out from the policy training set. In a match, each player will play as soloist against two copies of the opponent, as well as against two copies of itself. The baseline players are all versions of Kermit, with the only difference being the



(a) Grand

(b) Suit

(c) Null

(d) Null Ouvert

Fig. 1: Average *TSSR* after *Card Number* cards have been played. Data is separated by game type and whether the player to move is the soloist (left) or a defender (right).

inference module used. These inference modules are KI, CLI, with the addition of no inference (NI). This experiment is designed to see if (a) the performance of the player improves as measured by its play against opponents and (b) to determine the extent to which the defender and soloist performance is responsible for this difference.

For each match-up we report the average tournament points per game (TP/G) for the games in which the players played against each-other. The games in which the player played against a copy of itself were used to determine the difference in the effectiveness of the defenders and soloists.

$AvBB$ denotes a match-up in which the soloist is of type A while the defenders are both of type B. The value of the game $AvBB$ is in terms of the soloist score, therefore it is the sum of the soloist's score and the negation of the defenders' score. In this notation, the performance of player A relative to player B is given as

$$\Delta TP/G = [AvBB - BvAA]/3 \tag{4}$$

The value is divided by 3 since it is enforced that a player is soloist 1/3 the time in the tournament setup. To directly compare the performance of the defenders, we can measure the performance difference between scenarios where the only difference is the change in defenders.

$$\Delta Def/G = [(AvBB + BvBB) - (AvAA + BvAA)]/6 \tag{5}$$

A negative value for $\Delta Def/G$ indicates A performs better than B in defense. The same concept can also be applied to directly compare the efficacy of the soloist.

$$\Delta Sol/G = [(AvAA - BvAA) + (AvBB - BvBB)]/6 \tag{6}$$

A positive value for $\Delta Sol/G$ indicates A performs better than B as the soloist.

The results for the tournament match-ups are shown in Table III. All Δ values reported have a * attached if they are not found to be significant at a p value of 0.05 when a paired T-Test was performed. The general trend is that PI performs the best, followed by CLI, then KI, then NI. This fits with the expectation that better TSSR values seen in Figure 1 would translate into stronger game performance. Another interesting result is that the majority of the performance gain seems to be from the defenders, as demonstrated by the ΔDef values being consistently larger than the ΔSol values. The most interesting match-up is PI : CLI since it roots the previous state-of-the-art skat inference against the new policy-based method. PI outperforms CLI by 2.32, 0.64, and 1.57 TP/G in suit, grand, and null, respectively. The grand result did not provide statistical significance.

The major drawback of the PI inference is the runtime. When combined with evaluation, PI20 takes roughly 5 times longer to make a move than CLI.

Further experiments were conducted to test the effect of increasing the number of sampled worlds to 100,000 (PI100) and sampling states instead of card configurations (PIF20). In addition, a cheating version of Kermit was introduced (C) in which it places all probability on the true state. All programs were tested against CLI with only the mirrored adversarial setup used. The rest of the experimental setup was identical.

The results in Table IV indicate that PI100 performs stronger than the other PI variants in suit and grand, when playing against CLI, however, only the grand result is significant. The opposite is true for null, in which PI20 performs strongest out of the PI variants. This result contradicts the idea that a higher TSSR value necessarily corresponds to better cardplay performance. Cheating inference performs worse than CLI in all but null games. This is interesting because it puts all the probability mass on the true world, but still plays worse than a player that is not cheating. This result in conjunction with the worse null game score for PI100 indicates that further investigation into the exact role inference quality has within the context of PIMC is required. Also, PI20 outperforms PIF20 over all game types, showing that there can be benefits to sampling card configurations instead of states when there is a limited sampling budget.

One further experiment was performed to determine whether performance gains would be present with mixed defenders. This is interesting since it is possible the gain would only be present if the partner's inference was compatible with their own. For the sake of time, this was only done for the CLI and PI20 matchup. The added arrangements are $AvAB$, $AvBA$, $BvAB$, and $BvBA$. With these added, we now have six games for each tournament match. The results for this match-up are included in Table IV. PI is consistently stronger than CLI (the grand result is not significant), but the effect size is smaller. This is expected because PI is now defending against PI in the mixed setup games. To further analyze the the relative effectiveness of the players as soloist against only the mixed team defenders, we can calculate:

$$\Delta Sol = [(AvAB - BvAB) + (AvBA - BvBA)]/6 \tag{7}$$

ΔDef_B measures the difference in effectiveness of a mixed defense (A and B) and a pure defense of A's. It is calculated by averaging the effect of swapping in player B into defense for all match-ups that included two A's on defense. The reverse can be done to find the effect of swapping in A to form a mixed defense. A positive value for ΔSol means that PI is more effective than CLI as soloist in the mixed setting. A positive value for ΔDef_{PI} means defense improved when it was added, and same for ΔDef_{CLI}.

While all the values in Table V show the same trends of PI performing better on defense and soloist across all game types, the effect is only statistically significant for ΔSol and ΔDef_{CLI} in the suit games. These tests were done using pairwise TTests with a significance threshold of p=0.05.

V. Conclusion

Policy Inference (PI) appears to provide much stronger inference than its predecessors, namely Kermit Inference (KI)

TABLE III: Tournament results for each game type. Shown are average tournament scores per game for players NI (No Inference), CLI (Card-Location Inference), PI (Policy Inference), and KI (Kermit's Inference) which were obtained by playing 5,000 matches against each other, each consisting of two games with soloist/defender roles reversed. The component of ΔTP attributed to Def and Sol is also indicated

Game Type	Suit				Grand				Null			
Matchup	TP	Δ TP	Δ Def	Δ Sol	TP	Δ TP	Δ Def	Δ Sol	TP	Δ TP	Δ Def	Δ Sol
KI : CLI	17.62 : 20.81	-3.19	-2.76	0.42*	37.02 : 38.98	-1.96	-1.85	0.11*	17.22 : 19.83	-2.61	-2.70	-0.09*
KI : PI	16.48 : 21.61	-5.13	-4.22	0.91	36.44 : 39.56	-3.12	-2.30	0.83	17.29 : 19.66	-2.37	-2.93	-0.56*
NI : CLI	16.14 : 24.12	-7.98	-7.36	0.62*	36.56 : 40.45	-3.89	-3.44	0.46*	16.01 : 22.46	-6.45	-6.55	-0.10*
PI : CLI	19.50 : 17.18	2.32	1.64	-0.68*	37.87 : 37.23	0.64*	0.19*	-0.45*	18.84 : 17.27	1.57	1.14	-0.43*
KI : NI	23.29 : 18.64	4.65	4.53	-0.12*	39.71 : 38.36	1.35	1.30	-0.05*	21.65 : 17.81	3.84	3.85	0.01*
NI : PI	14.59 : 25.02	-10.43	-9.07	1.36	36.46 : 40.01	-3.55	-3.14	-0.42*	15.77 : 22.10	-6.33	-6.84	0.51*

TABLE IV: Tournament results for each game type. Shown are average tournament scores per game for players CLI (Card-Location Inference), PI20 (Policy Inference with 20,000 card configurations sampled), PIF20 (Policy Inference with 20,000 states sampled), PI20 (Policy Inference with 100,000 card configurations sampled), and C (Cheating Inference) which were obtained by playing 5,000 matches against each other, each consisting of two games with soloist/defender roles reversed.

Game Type	Suit		Grand		Null	
Matchup	TP	Δ TP	TP	Δ TP	TP	Δ TP
PI : CLI	19.50 : 17.18	2.32	37.87 : 37.23	0.64*	18.84 : 17.27	1.57
PIF20 : CLI	19.30 : 17.65	1.65	37.67 : 37.20	0.47*	18.01 : 17.66	0.35*
PI100 : CLI	19.55 : 16.69	2.86	38.19 : 36.09	2.10	18.13 : 17.10	1.03
C : CLI	14.75 : 18.00	-3.25	29.97 : 38.46	-8.49	20.80 : 10.99	9.82
PI : CLI (6way)	19.06 : 17.82	1.24	37.64 : 37.27	0.38*	18.31 : 17.74	0.57

TABLE V: Tournament results for each game type in the 6-way match between CLI and PI20. 5,000 matches were played for each game type.

Game Type	ΔSol	ΔDef_{CLI}	ΔDef_{PI}
Suit	1.00	-1.06	0.59*
Grand	0.38*	-0.02*	0.16*
Null	0.18*	-0.59*	0.55*

and Card Location Inference (CLI) as demonstrated by the TSSR value figures. Across the board, the higher TSSR values translate into stronger game-play as demonstrated in card-play tournament settings. PI20 outperforms CLI by 2.32, 0.64, and 1.57 TP/G in suit, grand, and null games, respectively. Also, it seems that increasing the number of states sampled increases the performance of PI, however, this did not translate into the null game type. Further investigation into this null game result is needed. We would expect that when substantially increasing the sampling threshold, the state sampling employed by PIF20 would be more effective. But under this limited sampling regimen, sampling card configurations is more effective than sampling states.

Future work related to inference in trick-taking card games should focus on the relationship between opponent modelling and exploitability. In order to investigate the robustness of our approach, we could try learning a best response to it. Likewise, adjusting player models online could enable us to better exploit our opponents and cooperate with teammates.

Another direction is to experiment with heuristics that allow our algorithm to prune states that are highly unlikely and stop considering them altogether. This could help us sample more of the states that are shown to be realistic given our set of human games and possibly improve the performance of the search.

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